[4]Bohm and Bub, Ref. 1.

[5]A good discussion of the photon as a two-state quantum system may be found in P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, Oxford, 1958), 4th ed., Chap. I.

[6]The linear polarizers are "HN-32 stripable polarizers," supplied by Polaroid Land Corporation, Cambridge, Massachusetts. The 15×10^{-4}-cm-thick polarizing material was epoxied onto optical flats. The index of refraction of the polarizing sheet is 1.5, resulting in a transit time of $\sim7.5\times10^{-14}$ sec.

[7]Since the extinction coefficient of the HN-32 polarizer is about 3×10^{-5}, it can be shown that ~90% of the photons entering the sheet interact in the first 3×10^{-4} cm of the polarizing sheet. This means that the distance between polarizers B and C should be taken as the distance between their front surfaces (surfaces facing the light source). When polarizers B and C are touching, this distance is just the thickness of polarizer B; i.e., 15×10^{-4} cm.

NEW SUM RULES AND SINGULARITIES IN THE COMPLEX J PLANE

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(Received 6 March 1967)

Exact sum rules to investigate singularities in the complex angular momentum plane are obtained.

Remarkable diffraction shrinkage at high energy for the reaction $\pi^-+p\to\pi^0+n$ has been successfully explained by the Regge-pole model based on a single ρ-meson exchange.[1-3] In addition, the dip phenomena observed in the above and other reactions have been clearly explained in the same model with a vanishing helicity-flip amplitude at $\alpha=0$.[4,5] On the other hand, the single-ρ-exchange model for the above reaction predicts no polarization, which is not consistent with the observed nonzero polarization.[6] Recently, some models, including a ρ' pole[7] or a cut[8] in addition to the ρ pole, were proposed to explain the above polarization. Theoretically, it has not been definitely proved yet whether there are other singularities like a ρ' pole or a cut with the same quantum numbers as the ρ meson in addition to the ρ pole in the complex-J plane.

The purpose of this Letter is to propose a new sum rule to obtain a definite answer for the above question. We consider the πp helicity-nonflip forward scattering amplitude with charge exchange,[9]

$$f^{(-)}(\nu)\equiv(4\pi)^{-1}[A^{(-)}+\nu B^{(-)}],\tag{1}$$

whose asymptotic behavior will be controlled by the ρ pole. Let us assume, at first, that there are no other singularities except the ρ pole in the complex-J plane for $\alpha_\rho\geqslant\alpha\geqslant-1$.[10] (No definite candidate is known among boson resonances with the same quantum numbers as those of the ρ, except on the ρ trajectory.)

Using the same technique[11] introduced by one of the authors, we separate $f^{(-)}(\nu)$ into the ρ-pole term $f_\rho(\nu)$ which behaves as ν^{α_ρ} at infinity, and the remaining term $f^{(-)\prime}(\nu)$ which vanishes faster than ν^{-1} at infinity due to our above mentioned assumption:

$$f^{(-)}(\nu)\equiv f_\rho(\nu)+f^{(-)\prime}(\nu).\tag{2}$$

Here we define

$$f_\rho(\nu)\equiv-\beta_\rho\frac{P_{\alpha_\rho}(-\nu/\mu)-P_{\alpha_\rho}(\nu/\mu)}{2\sin\pi\alpha_\rho}\tag{3}$$

with pion mass μ. Then, the dispersion relation for the $f^{(-)\prime}(\nu)$ is obtained as

$$f^{(-)\prime}(\nu)=\frac{g_r^2}{4\pi}\frac{\nu_B}{2m}\left(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right)+\frac{1}{\pi}\int_\mu^\infty d\nu'\left(\frac{1}{\nu'-\nu}-\frac{1}{\nu'+\nu}\right)\mathrm{Im}f^{(-)\prime}(\nu'),\tag{4}$$

$$\mathrm{Im}f^{(-)\prime}(\nu)=\frac{(\nu^2-\mu^2)^{1/2}}{4\pi}\tfrac{1}{2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]-\tfrac{1}{2}\beta_\rho P_{\alpha_\rho}(\nu/\mu),\tag{5}$$

and

$$\nu_B=\mu^2/2m.\tag{6}$$

In deriving Eq. (4), the crossing symmetry

$$\mathrm{Im}f^{(-)\prime}(\nu)=\mathrm{Im}f^{(-)\prime}(-\nu)\tag{7}$$

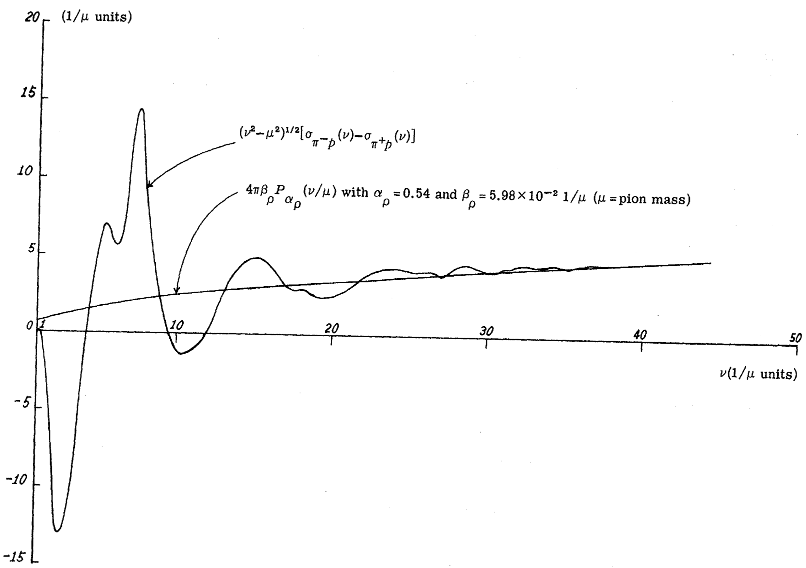

FIG. 1. The integrands of the sum rule (9) are plotted: The values of $(\nu^2-\mu^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]$ are taken from the experiments, and the values of $4\pi\beta_\rho P_{\alpha_\rho}(\nu/\mu)$ are calculated with $\alpha_\rho=0.54$ and $\beta_\rho=5.98\times10^{-2}\mu^{-1}$.

has been used.

Therefore, we are immediately led to the following sum rule of the superconvergent type, using Eq. (4) together with the property that $f^{(-)\prime}(\nu)<\nu^{-1}$ (for $\nu\to\infty$):

$$-\frac{g_r^2}{4\pi}\left(\frac{\mu}{2m}\right)^2+\frac{1}{\pi}\int_\mu^\infty d\nu' \,\mathrm{Im} f^{(-)\prime}(\nu')=0. \tag{8}$$

Thus, we finally obtain, from Eqs. (5) and (8),

$$4\pi f^2-(2\pi)^{-1}\int_\mu^\infty d\nu[(\nu^2-\mu^2)^{1/2}\{\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)\} -4\pi\beta_\rho P_{\alpha_\rho}(\nu/\mu)]=0, \tag{9}$$

with[12]

$$f^2=\frac{g_r^2}{4\pi}\left(\frac{\mu}{2m}\right)^2=0.081\pm0.002. \tag{10}$$

This sum rule (9) should hold if no J singularities extend above -1 at $t=0$, except for the ρ Regge pole.

In order to test the above sum rule (9), the following data were used: the πp total cross-section data tabulated by Barashenkov and Maltsev up to 1.6 GeV/c,[13] the data by the Moyer group[14] between 1.6 and 2.6 GeV/c, and the data by Citron et al. between 2.6 and 5.46 GeV/c.[15] We assumed that the Regge asymptotic behavior is already established at 5.46 GeV (=39.0 in units of pion mass) for the amplitude defined by Eq. (1).[2,16] For convenience, the first and second terms of the integrand of Eq. (9) are plotted in Fig. 1. In evaluating the second term, the value of $\alpha_\rho=0.54$ was chosen[2] as an example and the value of β_ρ was calculated by a least-squares fit to the total cross-section data above 6 GeV/c.[17] One can readily observe that the first term is approaching the second term (the Regge limit) at high energies, which insures the convergence of the integral. In Table I, we list the following two kinds of values of β_ρ with α_ρ ranging[18] from 0.53 to 0.59: (i) the calculated value of β_ρ assuming the sum rule (9) to hold and (ii) the value of β_ρ obtained by a least-squares fit using the data above 6 GeV/c.[17]

Therefore we can conclude as follows:

(i) Within the present accuracy of the total cross-section measurement, our sum rule (9) holds, which means that our assumption is consistent with the present data. Sufficiently accurate measurement (approximately up to the error ±0.02 mb) of the $\pi^\pm p$ total cross sections at high energy above 6 GeV will enable us to arrive at an almost definite conclusion regarding the above hypothesis.

(ii) At the present stage, however, we cannot rule out any possibility of the existence of a ρ' (or a cut) if the pole residue (or discontinuities) are reasonably small. If one has a finite number of poles $(\alpha_{\rho(i)}, \beta_{\rho(i)})$, with the same

Table I. Comparison of experimental and calculated values of β_ρ, respectively, with α_ρ ranging from 0.53 to 0.59.

α_ρ	The values of β_ρ determined from the data above 6 GeV/c ($10^{-2}\ \mu^{-1}$)	The values of β_ρ determined from the sum rule (9) ($10^{-2}\ \mu^{-1}$)
0.53	6.25 ± 0.50	5.66 ± 0.04
0.54	5.98 ± 0.47	5.54 ± 0.04
0.55	5.72 ± 0.45	5.36 ± 0.04
0.56	5.47 ± 0.43	5.20 ± 0.03
0.57	5.24 ± 0.41	5.06 ± 0.03
0.58	5.01 ± 0.39	4.88 ± 0.03
0.59	4.77 ± 0.38	4.72 ± 0.03

quantum numbers as those of the ρ meson, with α greater than -1, one can immediately extend the sum rule (9) to more general cases,

$$4\pi f^2-(2\pi)^{-1}\int_\mu^\infty d\nu[(\nu^2-\mu^2)^{1/2}\{\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)\} -4\pi\sum_i\beta_\rho(i)P_{\alpha_\rho(i)}(\nu/\mu)]=0. \tag{11}$$

Recently sets of values have been obtained for the ρ and ρ' parameters to explain both the differential cross section and the observed polarization in the π^-p charge-exchange scattering. These values of the ρ and ρ' trajectories, if the ρ' pole exists at all, must satisfy the above sum rule (11). However, the result is negative for these sets of values.[19] In the case of a cut,[8] the experimental check would be completely similar.

We hope that more extensive and accurate data on the total cross sections at high energies will soon be available so that the experimental check of our sum rules (9) and (11) can be made.

Detailed analysis including further applications will be published in a forthcoming paper.

Note added in proof.— After completing the manuscript, we found that A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze [Phys. Letters 24B, 181 (1967)] had also derived a sum rule similar to our sum rule (11).

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*Work supported in part by the Matsunaga Science Foundation.

[1]R. Logan, Phys. Rev. Letters 14, 414 (1965).

[2]R. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

[3]B. Desai, Phys. Rev. 142, 1255 (1966).

[4]F. Arbab and C. Chiu, Phys. Rev. 147, 1045 (1966).

[5]S. Frautschi, Phys. Rev. Letters 17, 722 (1966).

[6]P. Bonamy, P. Borgeaud, C. Bruneton, P. Falk-Vairant, O. Guisan, P. Sonderegger, C. Caverzasio, J. P. Guillaud, J. Schneider, M. Yvert, I. Mannelli, F. Sergiampietri, and L. Vincelli, Phys. Letters 23, 501 (1966).

[7]R. K. Logan, J. Beaupre, and L. Sertorio, Phys. Rev. Letters 18, 259 (1967).

[8]V. M. de Lany, D. J. Gross, I. J. Muzinich, and V. L. Teplitz, Phys. Rev. Letters 18, 149 (1967).

[9]We use the same notation as G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).

[10]It is important to note that we have no essential singularity at $\alpha=-1$ since we confine ourselves to the trajectory with negative signature.

[11]K. Igi, Phys. Rev. Letters 9, 76 (1962); Phys. Rev. 130, 820 (1963).

[12]W. S. Woolcock, in Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961 (Centre d' Etudes Nucléaires, Saclay, France, 1961). Vol. I, p. 459.

[13]V. S. Barashenkov and V. M. Maltsev, Fortschr. Phys., Sonderband 9, 549 (1961).

[14]M. J. Longo and B. J. Moyer, University of California Radiation Laboratory report No. UCRL-10174, 1962 (unpublished).

[15]A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. Rousset, and R. H. Sharp, Phys. Rev. 144, 1101 (1966).

[16]V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966).

[17]W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, and A. L. Read, Phys. Rev. 138, B913 (1965).

[18]According to Ref. 1, the least-square fitting analysis for the $\pi^\pm p$ total cross sections in the momentum range from 6 to 20 GeV/c gives us the value of $\alpha_\rho = 0.56 \pm 0.15$.

[19]Substituting the values $\alpha_\rho=0.58$, $\beta_\rho=4.06\times10^{-2}\mu^{-1}$ and $\alpha_{\rho'}=0.17$, $\beta_{\rho'}=9.93\times10^{-2}\mu^{-1}$, which correspond to the values of the parameters in model III obtained by Logan et al. in Ref. 4, the sum rule (11) gives $4\pi f^2 = -7.42 \pm 0.11$, while, according to Woolcock (Ref. 12), the experimental value is $4\pi f^2=+1.018 \pm 0.025$. The values of the parameters in model II give $4\pi f^2=-3.37 \pm 0.11$, which is also inconsistent with experimental value. Even though one introduces a ρ' trajectory to explain the observed polarization, one must choose those values of the parameters which are consistent with our sum rule (11).